Chapter 10 Brief introduction to spatial panel regression and SVC panel regression

*Learning Objectives*

1. *Understand* the concept and characteristics of panel data and the application of panel regression models.
2. *Distinguish* between fixed effects, random effects, and individual, time, and two-ways models in the context of panel regression.
3. *Analyze* the theoretical debate and statistical testing methods used to inform the choice between different panel regression models.
4. *Categorize* different spatial panel models, including spatial lag panel model, spatial error panel model, and spatial Durbin panel model.
5. *Identify* the two sources of spatial autocorrelation in panel regression residuals and understand how to apply the Lagrange Multiplier tests to choose the suitable model.
6. *Appreciate* the spatially varying coefficient process with panel models, including Geographically and Temporally Weighted Regression (GTWR), Geographically Weighted Panel Regression (GWPR), and Random Effect Eigenfunction Based Spatial Filtering SVC Panel Regression.
7. *Apply* the above models in R and interpret the output.

10.1 Panel data set and panel regression

A panel dataset—also known as longitudinal data—adds an essential dimension of depth to the observations by capturing changes over time, making it inherently multidimensional. This chronological layout brings with it the advantage of exploring dynamics that other types of data (the cross-sectional data) may overlook or simply lack the temporal depth to look at. With panel data, we can establish panel regression analysis. A general form of panel regression analysis follows this formula:

where *T* represents time and is an *NT* × 1 vector that stacks the dependent variables vectors at time 1 to *T*. *N* is the number of individuals. Other matrices and vectors with the subscript are defined similarly. **X** is the matrix of explanatory (independent) variables. captures the individual effects where is an *NT*×*KG* matrix of *KG* individual dummy variables indicating district, race, sex, and so on (Greene, 2003) and is a *KG*×1 coefficient vector. captures temporal effects where is an *NT*×*KH* matrix indicating time, such as day, month, and year. is a *KH*×1 coefficient vector. The static panel model is often estimated as an individual one-way model (namely, the term **H**1:*T***γ***H* is dropped) or a time one-way model (namely the term is dropped), or a two-ways model (both terms are kept).

A crucial strength of panel data lies in its ability to observe and account for individual or spatial unit-specific effects that persist over time () or time-specific effects that persist over individuals/spatial units (). These effects, whether they are individual characteristics in a population study or specific attributes of spatial units in a geographical analysis, or trends over time that influence all individuals, could lead to biased results if left unaccounted for in cross-sectional or pure time-series data analysis. For instance, in a socioeconomic study, unobserved characteristics like individual ambition or specific cultural norms might be constant over time but could significantly affect the observed outcomes. Similarly, in a geographical study, some inherent features of a location—like its topography or climatic patterns—might not change over time but could have substantial impacts on the relationships being studied. Furthermore, broad temporal trends like economic cycles or seasonal patterns could similarly shape the outcomes across different individuals or spatial units.

By leveraging the repeated observations of the same units over time, panel data enables us to explore both within-unit changes and between-unit variations when conducting regression analysis. Within-unit changes can capture the dynamic nature of the unit itself, such as how an individual’s income varies with age or how a city’s air quality evolves with its level of industrialization. This perspective is particularly valuable when considering external shocks or policy changes, as panel data allows for the examination of their diverse impacts across time and units.

Additionally, panel data facilitates exploration of between-unit variations, allowing for comparison of how different individuals or spatial units respond to similar conditions or stimuli over the same timeframe. For instance, how various cities react to an increase in federal infrastructure funding or how different demographic groups’ health status changes in response to a new health policy can be studied.

Moreover, panel data can help in identifying time-varying effects that impact all units under study. For example, the effects of a nation-wide policy change or a global economic recession would be temporally consistent across all spatial units and can be effectively examined using panel data.

The use of panel data augments the depth and breadth of analyses, leading to richer, more nuanced insights unattainable with cross-sectional or pure time series data alone. These insights can markedly enhance the robustness of the findings and the predictive power of the models built on these findings.

10.1.1 Fixed, random effects and individual, time and two-ways models

In the context of panel regression, the type of model we employ largely depends on two key considerations. First, we decide whether to model the effects (, or ) as fixed or random, which leads us to choose between fixed effects or random effects models. Second, we determine our analytical focus: are we examining the effects pertaining to individuals (only ), time periods (only ), or both? This distinction results in the six model types: individual fixed effects, individual random effects, time fixed effects, time random effects, two-ways fixed effects, and two-ways random effects models. While the two-ways models might seem appealing at first glance, capable of simultaneously handling both individual and time effects, it is crucial to weigh this against the increase in model complexity they introduce. This added intricacy can offset any potential advantages brought about by incorporating both individual and time effects, particularly when it comes to interpretability and the efficient use of degrees of freedom. Consequently, it is pivotal to make this decision with a clear understanding of your data, research question, and the balance between model sophistication and interpretability.

**Fixed Effects Models (FE)**: These models are constructed on the assumption that individual, or spatial unit-specific (or time-specific) effects (, or ) are correlated with the predictor variables. Essentially, the FE model posits that each individual or spatial unit (e.g., a person, a city, a country) or time has its own unique attributes that might affect the predictor variables. These attributes are constant over time and unique to each individual or unit—hence, “fixed effects.”

To illustrate, we will use the individual fixed effect (time fixed effect is similar). Suppose we are studying the effect of education level on income across different cities over time. Each city has its own unique characteristics, such as its industry mix, local tax policies, and cost of living, which could affect both education levels and income. An FE model allows us to control these unique, time-invariant characteristics by letting the intercept (the baseline level of income) vary for each city.

By doing this, the FE model focuses on estimating the effects of variables that change over time within each city, thus capturing the within-city variation. This model allows us to estimate the relationship between education and income while controlling for those unobserved city-specific factors.

**Random Effects Models (RE)**: Unlike FE models, RE models assume that individual or spatial unit-specific (or time-specific) effects (, or ) are not correlated with the predictor variables. In other words, the unique attributes of each individual or unit are viewed as random variables, drawn from a larger population, with a common mean.

Continuing with our example, an RE model would view the unique characteristics of each city (like industry mix, local tax policies, cost of living) as random variables independent of the city’ education levels. These city-specific random effects are assumed to influence the income level but are not directly related to the education level.

As such, the RE model can estimate the effects of both time-variant (or individual-variant) and time-invariant (or individual-invariant) variables, capturing both within-city and between-city variations. However, the crucial assumption here is that these city-specific effects are random and uncorrelated with the predictors. If this assumption is violated, the RE model could provide biased estimates.

**Two-Ways Models**: This type of model provides a more sophisticated framework, incorporating both individual or spatial unit-specific effects and time effects (both and are kept in the model). The assumption here is that, besides the unique characteristics of each individual or unit, there are also common effects that influence all individuals or units at specific times.

For instance, in our example, a Two-Ways model would consider both the unique attributes of each city and common time-specific effects, such as a nationwide economic recession or a change in federal education policy, that could affect all cities’ education levels and income at the same time. By accounting for these two types of effects, the Two-Ways model can provide a more comprehensive analysis of the relationship between the predictor and outcome variables in panel data, though in real life data analysis, the complexity of both individual and time level effects is likely entangled, which makes interpretation and understanding of the model rather impractical.

In deciding which model to choose, it largely depends on the nature of your data and the research question at hand. Statistical tests, such as the Hausman test, can provide guidance by testing whether the individual or spatial unit-specific effects are correlated with the predictor variables. If they are, an FE model might be more appropriate; if they are not, an RE model might be a better choice. The Two-Ways model is especially useful when there are reasons to believe both individual or spatial unit-specific effects and time effects play significant roles, and clear distinctions between the two effects can be obtained either theoretically or empirically.

10.1.2 Theoretical debate and statistical testing

Deciding between Fixed Effects and Random Effects models involves both theoretical considerations and statistical testing. On the theoretical front, we need to consider the substantive nature of the individual (or time)-specific effects in our model. In panel data analysis, individual or spatial (time)-specific effects refer to certain unique characteristics or attributes of the entities being observed (be it individuals, regions, or time periods) that might have an impact on the dependent variable (for example, GDP per capita in a regional economic study) but are not directly measured or included in our model as independent variables because we either are not aware of their existence or their data is not systematically collected.

If we expect these effects to be correlated with the predictors – meaning that the unobserved individual or time-specific effects are systematically related to the explanatory variables – a Fixed Effects model should be utilized. For instance, if we are studying the effect of urbanization on GDP per capita in a regional study, and we believe that there are unobserved characteristics specific to each spatial unit that are correlated with both urbanization and GDP per capita, a Fixed Effects model would be a suitable choice.

When we say that these effects are “correlated with the predictors” in the Fixed Effects (FE) model context, we mean that these unobserved individual (time) effects have a systematic relationship with our independent variables (like urbanization) that can potentially bias the estimation of our regression coefficients if not properly accounted for. FE models provide a way to control these unobserved effects by allowing the intercept to vary across the entities.

If we expect that these effects are not correlated with the predictors—that is, the unobserved individual or time-specific effects are random and not systematically related to the explanatory variables – a Random Effects model would be more fitting. This does not mean, however, that these effects do not impact the dependent variable (GDP per capita); it just means that they do not systematically relate to our predictors and thus do not introduce a bias into our estimations of the independent variables. The unobserved effects in RE models are treated as random variables with a common variance, which are included in the error term of the model.

We include the random effects in our model because, even if these effects are uncorrelated with the predictors, they could still introduce variability into our dependent variable that we want to account for. By including them in our model, we can get a more accurate picture of the total variability in our dependent variable (GDP per capita) and thus improve the precision of our estimates.

It is also worth noting that the choice between an FE and an RE model can often depend on the specifics of your dataset and research question. FE models are usually preferred when you are interested in the impact of variables that change over time, while RE models can be more efficient when you are dealing with variables that are constant over time or when you are interested in making inferences about a population based on a sample.

On the empirical front, statistical testing is another important aspect to consider when deciding between these models. A commonly used statistical test in this context is the Hausman Test. This test checks the null hypothesis that the individual-specific effects are uncorrelated with the predictors. If this hypothesis is rejected, it suggests that the individual-specific effects and the predictors share a systematic relationship, implying the suitability of a Fixed Effects model. On the other hand, if we fail to reject the null hypothesis, it suggests the absence of a systematic relationship, making the Random Effects model a more fitting choice. The following code uses the Greater Beijing Area data to construct a panel dataset, then conducts both fixed and random effects panel regression. A Hausman Test is performed to suggest which model might be appropriate for the data.

# Set you work space

setwd("/yourworkspace/")

# load necessary libraries:

library(spdep)

library(sp)

library(ggplot2)

library(spatstat)

library(classInt)

library(dplyr)

library(plm)

library(splm)

library(reshape2)

library(sf)

library(tidyverse)

spatial\_data <- st\_read(".", "gblntrans")

# Arrange spatial\_data by 'COUNTYNAME' alphabetically, this is critical for panel data analysis because the panel data transformation will re-arrange the spatial units alphabetically:

spatial\_data <- spatial\_data %>% arrange(COUNTYNAME)

spatial\_coord <- st\_coordinates(st\_centroid(spatial\_data))

# Extract only the needed data for panel regression:

spatial\_data\_df <- spatial\_data\_df[, c("COUNTYNAME", "GDPPC95","GDPPC96","GDPPC97","GDPPC98","GDPPC99","GDPPC00","GDPPC01", "FININCPC95","FININCPC96","FININCPC97","FININCPC98","FININCPC99","FININCPC00","FININCPC01", "FDIPC95","FDIPC96","FDIPC97","FDIPC98","FDIPC99","FDIPC00","FDIPC01", "FIXINVPC95","FIXINVPC96","FIXINVPC97","FIXINVPC98","FIXINVPC99","FIXINVPC00","FIXINVPC01", "URB95","URB96","URB97","URB98","URB99","URB00","URB01")]

# Reshape the data to long format

spatial\_data\_long <- spatial\_data\_df %>%

pivot\_longer(-COUNTYNAME,

names\_to = c(".value", "Year"),

names\_pattern = "([A-Za-z]+)([0-9]+)",

values\_drop\_na = TRUE)

# Correct the year values

spatial\_data\_long$Year <- ifelse(as.numeric(spatial\_data\_long$Year) < 50,

paste0("20", spatial\_data\_long$Year),

paste0("19", spatial\_data\_long$Year))

# Creating a pdata.frame for panel data analysis

spatial\_data\_p <- pdata.frame(spatial\_data\_long, index = c("COUNTYNAME", "Year"))

# Set the formula

gb.fm.p <- GDPPC ~ FININCPC + FDIPC + FIXINVPC + URB

# Fit Fixed Effects and Random Effects models

fe\_model <- plm(gb.fm.p, data = spatial\_data\_p, model = "within")

re\_model <- plm(gb.fm.p, data = spatial\_data\_p, model = "random")

# Conduct Hausman Test between Fixed Effects and Random Effects Models

hausman\_test <- phtest(fe\_model, re\_model)

# Print models summaries

summary(fe\_model)

summary(re\_model)

# Print Hausman Test result

print(hausman\_test)

The Hausman test, in this context, is used to decide between a Fixed Effects (FE) and a Random Effects (RE) model. The null hypothesis of the Hausman test is that the preferred model is random effects (either , or is not correlated with the independent variables), while the alternative hypothesis is that the preferred model is fixed effects.

Here, the p-value is less than 0.05 (in fact, it is so small that it rounds to 2.2e-16, which is the smallest number the system can construct), which indicates that the chance of making a Type I error is very slim. Therefore, we reject the null hypothesis that the preferred model is the Random Effects model. Instead, we conclude that the Fixed Effects model is the preferred one. This result suggests that there is a significant correlation between the unobserved individual effects and the regressors in the model, and thus it is more appropriate to treat these effects as fixed rather than random.

The selection between individual effects, time effects, and two-way effects models (which one of , or to drop, or not to drop at all) in panel data analysis also involves both theoretical considerations and empirical tests. However, unlike the choice between fixed effects and random effects models where statistical tests like the Hausman test are often adequate, the choice between these three types of models often relies more heavily on the theoretical considerations and the nature of the specific research question and dataset at hand.

On the theoretical front, if the nature of your research question suggests that unobserved characteristics associated with the entities (individuals, regions, etc.) could be affecting the dependent variable, then you should consider using an individual effects model. These models control for any unobserved, time-invariant characteristics that are unique to each entity but are not directly included in the model. This is often used when you believe that differences across entities (rather than time) are the primary source of variations in your dependent variable. For instance, in a study examining the effect of policy changes on economic growth across different countries, individual effects might include cultural, geographical, or historical characteristics that are specific to each country and do not change over the study period.

Conversely, if you believe that some sources of variation in your dependent variable arises from variations over time rather than across entities, a time effects model may be more appropriate. Time effects models control for unobserved, entity-invariant characteristics that vary over time. For example, in a study examining the effect of financial development on income inequality within a single country over multiple years, time effects could capture nationwide changes such as macroeconomic shocks, policy changes, or global economic trends that are not directly included in the model.

If, however, you believe that both time-invariant individual characteristics and entity-invariant temporal characteristics could be affecting your dependent variable, then a two-way effects model would be the most appropriate choice. These models allow you to control for both types of unobserved effects simultaneously. For example, in a panel data analysis of the effects of education and health expenditures on economic growth across various countries over multiple years, both country-specific effects (such as political system, culture, or geographical location) and year-specific effects (such as global economic shocks or worldwide technological trends) may be relevant to the variation of the dependent variable. This conceptual delineations are well reflected in the general panel regression formula.

In general, the choice between these models often comes down to careful consideration of your research question, your theoretical framework, and your understanding of the dataset. You should consider the nature of the unobserved effects that you believe are present, this requires thorough understanding of the theoretical foundation of the research question. You should also consider the specific entities and time periods that your data covers, and the potential sources of variations in your dependent variable. Running different models and comparing the results can also provide useful insights. However, it is essential to remember that each of these models makes specific assumptions about the nature of the unobserved effects, and the validity of these assumptions should be critically evaluated based on your understanding of the research context.

Empirical testing for choosing between individual effects, time effects, or two-way effects models is less straightforward than the decision between fixed and random effects models. Often, the choice of the model depends more on the structure of the data and theoretical considerations rather than statistical testing.

However, you can apply F-tests or likelihood ratio tests to evaluate the significance of individual and time effects in the context of a specific model. In R, these tests can be conducted using the plmtest() function in the “plm” package. This F-test or likelihood ratio test essentially test the presence of individual effects versus a pooled OLS model and time effects versus a pooled OLS model.

In the context of panel data, a pooled OLS model is a simple extension of the standard OLS model that does not take into account any potential individual or time effects. It treats the data as a simple cross section where all observations are assumed to be independent of each other.

In this sense, the pooled OLS model does not differentiate between individuals and does not consider the panel structure of the data, i.e., the fact that multiple observations are linked to the same individual or group over time. Instead, it simply pools all observations together and estimates the regression model as if the data were one large cross section.

While pooled OLS is easy to estimate and interpret, it can lead to biased and inefficient estimates if there are unobserved individual or time effects that influence the variation of the dependent variables. Such effects are common in panel data, where multiple observations per individual or group are likely to share certain unobserved characteristics (such as inherent abilities in a wage study, or local traditions and norms in a regional study). This makes the pooled OLS model a good candidate as a Null Hypothesis that there is neither individual nor time effects.

The following R script estimates a pooled OLS model first, and then tests the individual and time effects using the plmtest() function.

# Testing for individual and/or time effects:

# Estimate a pooled OLS model

pool.mod <- plm(gb.fm.p, data = spatial\_data\_p, model = "pooling")

# Test for individual effects

individual\_effects\_test <- plmtest(pool.mod, effect = "individual")

# Test for time effects

time\_effects\_test <- plmtest(pool.mod, effect = "time")

# Test for Two-Ways:

twoways\_effects\_test <- plmtest(pool.mod, effect = "twoways")

# print test results

print(individual\_effects\_test)

print(time\_effects\_test)

print(twoways\_effects\_test)

All the tests have a null hypothesis of no individual/time/two-ways effects. If the p-value is significant for the individual effects test, you would reject the null hypothesis of no individual effects and consider using a model that accounts for individual effects. Similarly, if the p-value is significant for the time effects test, you would reject the null hypothesis of no time effects and consider using a model that accounts for time effects, and the same goes for the two-ways effects.

However, these tests do not provide a direct way to choose between individual effects, time effects, and two-way effects models. They only indicate whether individual or time effects are present in the data. This is especially troublesome when at least two of the tests come out significant, which is quite often in empirical studies (as seen in this example, both the individual effects and two-ways effects tests come out significant). Therefore, the decision should still be primarily based on theoretical considerations and the nature of the dataset. In some cases, for instance, you may choose to use a two-way effects model even if only the individual or the time effects test is significant if you have a theoretical reason to expect both types of effects to be important, this is particularly true when you have a long and wide panel data set (covers many time periods and spatial units).

10.2 Spatial panel models

In Chapter 8, we introduced spatial autoregressive models in the cross-sectional data analysis as a response to the spatial autocorrelation existing in regular cross-sectional regression’s residuals, which emerges when analyzing spatial data. Spatial autocorrelation arises when the regression residual at a particular location is not independent from residuals at neighboring locations. This is usually a result of the endogenous interaction and/or correlated effects, either because of the spatially autocorrelated dependent variables (endogenous interaction effects), or because of missing spatially autocorrelated independent variables (correlated effects) (Elhorst, 2014).

Just as with cross-sectional spatial autoregressive models, spatial panel models are also a response to the presence of spatial autocorrelation inevitable in the regular panel regression residuals. Traditional panel data models, just like traditional cross-sectional regression, often do not consider spatial autocorrelation in its residuals. This omission can result in biased and inefficient estimates, given that spatial effects are universal for spatial data, and spatial autocorrelation in regression residuals is inherent. For instance, the economic performance of a region may be influenced not just by its own policies and characteristics, but also by those of its neighboring regions. This holds true for both cross-sectional and panel data analysis.

While panel data analysis is more complex because of the added temporal dimension, the impacts of spatial effects on panel data analysis are similar as in the cross-sectional regression analysis (Elhorst, 2014). Even with the presence of the many different specifications regarding the additional effects that now present because of the added dimension, being it individual, time, two-ways, or fixed, random, there will still be three types of interactions resulted from the spatial effects, namely, the endogenous interaction effects, the correlated effects, and the exogenous interaction effects. Similarly, as with cross-sectional regression analysis, the exogenous interaction effects will not cause the model’s residuals to be spatially autocorrelated, hence will not be considered as a spatial data model. Per Elhorst (2014), a full panel model considering all three types of interactions can be written as (only one time snap of the model is written here):

where is the dependent variable at time (we are presenting only a time snap from the general panel regression model for simplification purpose). is the endogenous interaction, also called the spatial lag. is a spatial weight matrix that defines the neighboring relationship among states. is the coefficient for the spatial lag. is the matrix of predictor variables, is the vector of coefficients of the predictors. is the exogenous interaction (also called the Durbin term, as we have introduced in Chapter 8), and the vector of its coefficients. is the error term. is the correlated effects (error interaction), and its coefficient. is the independent and identically distributed (i.i.d.) random noise. Because it nests all three interaction effects, this model is called General Nesting Spatial (GNS) model (Elhorst, 2014).

In addition, it is important to note a key distinction between individual effects () in a panel model and spatial effects (,,or ). While both effects relate to the individual spatial units’ characteristics, their origins and influences significantly differ.

Spatial effects are essentially the results of the First Law of Geography. This law encapsulates the principle of spatial dependence, meaning that values observed at nearby locations are not independent but instead tend to be similar. This similarity often follows a distance-decaying process, with the strength of the relationship diminishing as the distance between the locations increases. For example, in an analysis of property prices, the spatial effect would reflect how prices at one location might be influenced by the prices of nearby properties due to factors like shared neighborhood characteristics or local market trends.

Individual effects in panel data analysis, on the other hand, are distinct attributes or characteristics inherent to the individual spatial units that remain constant over the time span of the analysis. These effects capture the unique features of each unit that can influence the variable of interest (dependent variable), separate from any spatial interaction effects. For instance, in the case of regional economic analysis, individual effects might encapsulate aspects such as the region’s industry composition, educational level, or historical development. These aspects shape the region’s economic performance in ways that are intrinsic to the region itself, irrespective of its geographical neighbors.

Therefore, while spatial and individual effects might seem similar at first glance due to the fact that they all pertain to individual spatial units, they represent different types of influence. Spatial effects embody the spatial interaction and spatial spillover between units, following a distance-decaying process, while individual effects represent the distinct, time-invariant characteristics of each spatial unit. This is why the GNS model needs to write them in separate terms. Proper understanding of these effects is vital in model specification, as their treatments are quite different in panel data analysis. Recognition of the existence of spatial effects is the very reason for the implementation of spatial panel regression models in the first place.

10.2.1 Two sources for spatial autocorrelation in panel regression’s residuals

Just as in cross-sectional analysis, spatial autocorrelation in panel regression residuals can arise from two sources: spatial lag dependence (), which is the result of endogenous interaction effects, and spatial error dependence (), which is the result of correlated effects.

For spatial lag dependence, it occurs when the variable we are interested in (dependent variable) for a certain spatial unit is influenced by the same variable in nearby units (spatial autocorrelation of the dependent variable). This effect represents the inherent interaction that exists between neighboring entities. For instance, consider the economic performance of regions. The GDP of a region is not just determined by its economic contributing factors, such as local policies or workforce capabilities, but is also influenced by the GDP of its neighboring regions. Per the endogenous interaction, a prospering region has a higher chance to boost the economic activity of its neighbors through trade, labor movement, shared infrastructure, and other types of cross-regional interactions. These are often referred to as “spillover effects” and is common in many social science studies. The spatial lag model, which incorporates these endogenous interactions, is thus an important tool in our spatial data analysis arsenal.

On the other hand, spatial error dependence arises when unobserved factors, which affect the dependent variable, are spatially autocorrelated. These might be elements not included in our model but are geographically distributed in a manner that induces a spatial structure in the model’s errors. For instance, regional cultural similarities, political climate, environmental factors, or historical legacies, which might be challenging to measure or include in our model, could be spatially autocorrelated. When these omitted variables influence our dependent variable, they cause spatial correlation in the model’s error terms. The spatial error model, which accounts for this type of spatial autocorrelation, allows us to tackle such situations.

Understanding these two sources of spatial autocorrelation — spatial lag dependence and spatial error dependence — is essential. They represent the spatial structure and dependencies in our data, and recognizing their presence guides us in the selection of appropriate spatial panel models to analyze our data effectively and accurately. The subsequent sections will delve deeper into the models developed to address each type of spatial autocorrelation, their characteristics, and their applications.

10.2.2 Spatial lag panel model

The Spatial Lag Panel Model (SLPM) is a panel model specification that is designed to address the endogenous interaction effects (is assumed to be either not there or not as important). At its core, the SLPM is built upon the premise that a variable of a region is not only defined by its own contributing factors but is also shaped by the same variables of its surrounding regions. It postulates that the dependent variable for a specific observation is influenced by the same variable for neighboring observations, thereby encapsulating the spatial spillover effect. This effect represents the way in which changes in one location can propagate and induce changes in nearby locations, reflecting the inherent interconnectedness of the spatial units under consideration.

The spatially lagged dependent variable incorporated in the SLPM represents this spillover effect. This variable is generally calculated as a multiplication of a spatial weight matrix and the dependent variable, reflecting the weighted average of the dependent variable values for the neighboring regions.

The spatial weights matrix, a critical component of the SLPM, delineates the structure of spatial relationships in the data. It quantifies the notion of “neighborhood” or “proximity” among the observations, effectively embodying the geographical context within the model. We have extensively discussed the spatial weight matrix in Chapter 4, the details will not be repeated here. Suffice it to say that this spatial weight matrix plays a central role in incorporating spatial effects in the modeling framework.

By accommodating this spatially lagged dependent variable as an explanatory variable in the model, the SLPM adjusts for the possible influence of neighboring units on each observation, thereby explicitly incorporating spatial interaction effects in the model structure. Consequently, the SLPM allows for a more nuanced analysis of spatial processes where regional outcomes are interdependent, capturing both local dynamics and wider spatial influences.

Under the SLPM specification, the individual, time, or two-way effects, as well as fixed or random effects, can all be considered theoretically. However, the implementation of time or two-way random effects models poses certain challenges.

The key reason behind this complexity is the difficulty in estimating the model with random effects across both spatial and temporal dimensions simultaneously. More specifically, the estimation of these models requires an assumption of uncorrelated random effects across both time and space, which might not hold true in many real-world scenarios. For instance, if there are unobserved characteristics that vary over time but are correlated across different spatial units (or vice versa), this would violate the assumption of independent random effects.

Additionally, the estimation process for these types of models can be computationally intensive and require large amounts of temporal data, which may not always be available. This complexity can potentially lead to convergence issues and unstable estimates, limiting the practical utility of such models in certain cases.

It is also worth noting that the interpretation of the results from a time or two-way random effects model can be challenging. In particular, separating the influence of individual-specific random effects from time-specific random effects can be difficult and may require strong assumptions. For these reasons, while time or two-way random effects models can be considered under the spatial lag panel model specification in theory, they are not readily implemented in practice. It is typically more feasible and straightforward to focus on individual fixed or random effects models, or to consider time fixed effects or a two-way fixed effects model, which avoids some of these complexities.

The following script uses the Greater Beijing Area’s data to estimate a spatial panel lag individual fixed effect lag model. Spatial lag time fixed effect, two-ways fixed effect, or spatial lag individual random effect panel model can all be estimated by altering the key parameters in the spml() function from the splm package. In particular, by switching the “model=c("within","random","pooling"),” and “effect=c("individual","time","twoways")” we are able to specify fixed effect (model = "within"), random effect (model = "random"), individual effect (effect="individual"), time effect (effect="time"), or two-ways effect (effect="twoways"). In addition, for the spatial panel lag model, it is critical to make sure that both the switches lag = TRUE and spatial.error = "none" are on.

# 1. Spatial individual fixed effect model (lag model):

# Load the splm library if it is not yet loaded:

library(splm)

# Create the weight matrix list:

gb.listw <- nb2listw(poly2nb(spatial\_data))

# The lag switch must be TRUE and spatial.error must be set to be "none" so the spml() will return a spatial lag panel model.

# Otherwise, it will return a spatial autocorrelation panel model.

gb.splm.individual.fixed <- spml (gb.fm.p, data = spatial\_data\_p, listw = gb.listw, model = "within", effect = "individual", lag = TRUE, spatial.error = "none")

# Summarize the result, pay attention to the spatial lag parameter:

summary(gb.splm.individual.fixed)

10.2.3 Spatial error panel model

The Spatial Error Panel Model (SEPM) is another specification of spatial panel model. It addresses the inherent spatial autocorrelation present in the error terms of regression models directly instead of treating it as a result of the dependent variable’s spatial autocorrelation ( is assumed not there or not as important).

In a nutshell, SEPM operates on the premise that unexplained variations in the dependent variable at one location could be related to unexplained variations at nearby locations (they are unexplained because there are significant independent variables missing, and no practical ways to acquire them). This is embodied in the assumption that the regression model’s residuals (or error terms) - which encapsulate these unexplained variations – exhibit spatial autocorrelation. This means that if we observe an unusually high or low residual (representing a higher or lower observed value than our model predicts) at one location, we are likely to observe similarly high or low residuals at neighboring locations. In other words, the error terms are spatially clustered. This spatial clustering of error terms indicates that some vital explanatory factors, which have spatial characteristics, are missing from our model.

Let’s illustrate this with an example. Suppose we are analyzing the effect of local government spending on education outcomes across various school districts. While we might include variables like the amount of spending per student and socio-economic indicators, there could be other factors that we cannot measure or do not have data for, such as parental involvement or quality of teaching. If these unobserved factors are geographically clustered, and they usually are – maybe due to cultural similarities or policies in nearby districts – our model’s error terms will exhibit spatial autocorrelation. The SEPM takes into account this spatial structure of the unobserved factors.

On a similar note, the importance of SEPM is particularly highlighted in scenarios where we suspect the presence of general spatial spillovers, or the influence that unobserved factors in one region have on another region. Consider an example where we are studying the impact of tax policies on business growth across multiple cities. Unobserved factors like business culture or entrepreneurial spirit might not be easily measurable, yet they can significantly impact our dependent variable (business growth) and are likely to spill over across neighboring cities due to inter-city business interactions. The SEPM enables us to account for this spatial contagion effect in the unobserved factors, thereby enriching our understanding of the spatial processes at play.

Remember, though, that using the SEPM does not mean we are capturing all the unobserved, spatially dependent factors directly in our model. Instead, we are acknowledging their existence and influence, and we are ensuring our model accounts for the spatial structure they create in our error terms. This enables us to have more confidence in the estimates and inferences we make based on our model.

The following script uses the Greater Beijing Area’s data to estimate a spatial error panel individual fixed effect lag model. Spatial error time fixed effect, two-ways fixed effect, or spatial error individual random effect panel model can all be estimated by altering the key parameters in the spml() function from the splm package. In particular, by switching the “model=c("within","random","pooling"),” and “effect=c("individual","time","twoways")” we are able to specify fixed effect (model = "within"), random effect (model = "random"), individual effect (effect="individual"), time effect (effect="time"), or two-ways effect (effect="twoways"). For the spatial error panel model, it is critical to make sure that the switches lag = FALSE and spatial.error = "b" or "kkp" are on. The "b" or "kkp" refers to different error definitions. The "b" parameter is proposed in Baltagi et al. (2003), which considers the spatial autocorrelation exists only in the error components. The "kkp" parameter is discussed in Kapoor et al. (2007), which assumes that spatial autocorrelation applies to both the individual effects and the remainder error components. Depending on the research purpose and theoretical understanding of the research problem, an appropriate spatial error structure is important for better understanding of the data and stories behind the data. For the Greater Beijing Area, we do believe that the individual effects might also be spatially autocorrelated, hence a "kkp" error specification is used. When you are using your own data, your understanding of the data and the research question will dominate which parameter to choose from.

# 2. Spatial individual fixed effect model (error model):

# Load the splm library if it is not yet loaded:

library(splm)

# Create the weight matrix list (if not yet created):

gb.listw <- nb2listw(poly2nb(spatial\_data))

# The lag switch must be FALSE and spatial.error must be set to be either "b" or "kkp" so the spml() will return a spatial error panel model.

gb.spem.individual.fixed <- spml (gb.fm.p, data = spatial\_data\_p, listw = gb.listw, model = "within", effect = "individual", lag = FALSE, spatial.error = "kkp")

# Summarize the result, pay attention to the spatial lag parameter:

summary(gb.spem.individual.fixed)

10.2.4 Spatial lag and error panel model (spatial autocorrelation panel model)

Some studies may require considering both spatial lag and spatial error simultaneously (so that and are assumed to present simultaneously), which is quite often the case that the spatial autocorrelation in the regression residuals is a result of both spatial spillover effects of the dependent variable and unobserved or missing independent variables. This situation gives rise to the Spatial Autocorrelation Panel Model (SAPM), which includes both spatially lagged dependent variables and spatially autocorrelated error terms in the model. However, as we detailed in Chapter 8, while this is likely the case, the identifiability problem because we cannot identify exactly which sources cause how much spatial autocorrelation in the residuals often prevent a meaningful spatial autocorrelation panel model to be easily interpreted.

Still, this dual incorporation might be necessary in situations where we have clear theoretical guidance or empirical evidence that interpretation of the two sources will not be ambiguous. For instance, if we have prior knowledge or secondary data indicating the presence and relative magnitude of both spatial spillover effects and spatially dependent unobservables, it may be justifiable to use the SAPM and interpret the results accordingly. In such cases, the SAPM can offer a richer and more nuanced understanding of the spatial dynamics underlying the phenomenon of interest.

Consider, for example, we are studying the impact of renewable energy policies on greenhouse gas emissions across several cities. Our dependent variable is the level of greenhouse gas emissions in each city, and one of our key independent variables is the stringency of the renewable energy policy.

In this context, the spatially lagged dependent variable could capture the influence of emissions in neighboring cities. For example, cities close to each other might share a regional power grid; therefore, emission levels in one city might directly influence those in neighboring cities.

At the same time, there could be unobserved factors that influence emissions and are spatially autocorrelated. These might include factors such as the general environmental consciousness of a city’s population, the strength of local environmental non-government organizations (NGOs), or regional weather patterns affecting the efficiency of renewable energy sources. These unobservables could be spatially autocorrelated, with nearby cities having similar levels of environmental consciousness, NGO activity, or weather patterns.

Now, let’s say we have secondary data or existing research that suggests the presence and relative magnitude of both these spatial processes – the influence of neighboring cities’ emissions (spatial spillover) and the similarity of unobserved local characteristics (spatially dependent unobservables). For instance, we might have survey data showing similar levels of environmental consciousness among cities that are geographically close or historical records indicating substantial inter-city influence in emission levels. In this case, it becomes justifiable to use the Spatial Autocorrelation Panel Model (SAPM) to account for both processes. The secondary data or existing research can guide the interpretation of the spatial parameters in the SAPM, helping us discern how much of the spatial autocorrelation in the residuals is due to spatial spillover effects and how much is due to spatially dependent unobservables.

We must bear in mind that applying a SAPM requires a solid theoretical justification for the simultaneous presence of both spatial lag and spatial error effects. Nevertheless, when these conditions are met, the SAPM can be a powerful tool for uncovering the complex spatial processes underlying the data.

The following script uses the Greater Beijing Area’s data to estimate a spatial autocorrelation panel individual fixed effect model. Spatial autocorrelation time fixed effect, two-ways fixed effect, or spatial autocorrelation individual random effect panel model can all be estimated by altering the key parameters in the spml() function from the splm package. In particular, by switching the “model=c("within","random","pooling"),” and “effect=c("individual","time","twoways")” we are able to specify fixed effect (model = "within"), random effect (model = "random"), individual effect (effect="individual"), time effect (effect="time"), or two-ways effect (effect="twoways"). For the spatial autocorrelation panel model, it is critical to make sure that the both the switches lag = TRUE and spatial.error = "b" or "kkp" are on.

# 3. Spatial individual fixed effect model (autocorrelation model):

# Load the splm library if it is not yet loaded:

library(splm)

# Create the weight matrix list:

gb.listw <- nb2listw(poly2nb(spatial\_data))

# The lag switch must be TRUE and spatial.error must be set to be "kpp" so the spml() will return a spatial autocorrelation panel model.

gb.sapm.individual.fixed <- spml (gb.fm.p, data = spatial\_data\_p, listw = gb.listw, model = "within", effect = "individual", lag = TRUE, spatial.error = "kkp")

# Summarize the result, pay attention to the spatial lag parameter:

summary(gb.sapm.individual.fixed)

10.2.5 The Lagrange Multiplier tests to choose the alternative

The choice between the spatial panel models – the Spatial Lag Panel Model (SLPM), the Spatial Error Panel Model (SEPM), and the Spatial Autocorrelation Panel Model (SAPM) – largely depends on the nature and source of the spatial autocorrelation present in the data, and whether such sources are identifiable. Yet just as in the cross-sectional spatial autoregressive analysis, to make an informed choice, we can also apply the Lagrange Multiplier (LM) tests.

The LM tests provide a systematic approach to discern the nature of spatial autocorrelation in the residuals. Also as in cross-sectional analysis, the LM tests can be either of the conditional type, meaning the test only allows for the presence of either endogenous interaction (assuming the correlated effects do not exist) or correlated effects (assuming the endogenous interaction effects do not exist), or they can be of the locally robust types, meaning the tests can test either endogenous interaction effects or correlated effects assuming the other type of effects also exist. We will not delve into the technical details and algebraic operations of the LM tests in this text, but interested readers are strongly encouraged to consult Chapter 10 in Croissant and Millo (2019) for detailed discussions.

The model selection rules are also similar to the rules we presented in Chapter 8: For the conditional types, if one of the LM tests is significant and the other is not, then we would favor or choose the one that presents a significant LM test. If, however, both conditional LM tests are significant, then we will have to check the locally robust types. The p-values of the locally robust LM test will point to the alternative specification: if only one is significant, then we would favor the one that presents a significant robust LM test. If both p-values are significant, then the one with lower p-values (meaning a lower chance of making a Type I error) will be our chosen alternative. The following script uses the slmtest() function in the splm package for this test. Data used is the Greater Beijing Area data.

# LM test for spatial lag or spatial error models:

# Fixed effect models

lag.test.fixed <- slmtest (gb.fm.p, data = spatial\_data\_p, listw = gb.listw, model = "within", test = "lml")

error.test.fixed <- slmtest (gb.fm.p, data = spatial\_data\_p, listw = gb.listw, model = "within", test = "lme")

rlag.test.fixed <- slmtest (gb.fm.p, data = spatial\_data\_p, listw = gb.listw, model = "within", test = "rlml")

rerror.test.fixed <- slmtest (gb.fm.p, data = spatial\_data\_p, listw = gb.listw, model = "within", test = "rlme")

# Show the results:

lag.test.fixed

error.test.fixed

rlag.test.fixed

rerror.test.fixed

# Random effect models:

lag.test.random <- slmtest (gb.fm.p, data = spatial\_data\_p, listw = gb.listw, model = "random", test = "lml")

error.test.random <- slmtest (gb.fm.p, data = spatial\_data\_p, listw = gb.listw, model = "random", test = "lme")

rlag.test.random <- slmtest (gb.fm.p, data = spatial\_data\_p, listw = gb.listw, model = "random", test = "rlml")

rerror.test.random <- slmtest (gb.fm.p, data = spatial\_data\_p, listw = gb.listw, model = "random", test = "rlme")

# Show the results:

lag.test.random

error.test.random

rlag.test.random

rerror.test.random

10.2.6 Spatial Durbin panel model (not a spatial model)

The Spatial Durbin Panel Model (SDPM), although sometimes referred to as a spatial model, does not account for spatial autocorrelation directly. Rather, it incorporates spatially lagged independent variables in addition to the spatially lagged dependent variable in the model. This is the result of the exogenous interaction effects (only the terms is assumed to be present).

However, also as in the cross-sectional analysis, the spatial Durbin panel model can be combined with spatial lag and spatial error panel model to form spatial lag Durbin panel model or spatial error Durbin panel model, to account for either endogenous and exogenous interaction effects, or exogenous interaction and correlated effects.

In many social science or regional economic studies, incorporating the Durbin term (exogenous interaction effects) in addition to the spatial lag or spatial error terms might make sense because it suggests that not only the independent variables at the same location will cast influence on the dependent variable, but also the independent variables at neighboring locations will be influential as well.

For instance, in a study examining how per-capita income (dependent variable) in a given region is affected by factors such as education level and employment rate (independent variables). In a typical panel regression model, we would only consider the education level and employment rate within the same region when predicting per-capita income.

However, in a spatial Durbin panel model (in addition to spatial lag or spatial error), we would also consider the education levels and employment rates in neighboring regions. The reasoning behind this approach is the recognition that socio-economic factors do not exist in isolation; they are part of a complex, interconnected network. For instance, a region surrounded by areas with high education levels and employment rates could see an influx of educated workers or benefit from spillover effects of successful business practices, which in turn, could raise its per-capita income. This spatially lagged independent variable – the education levels and employment rates in neighboring regions – is what we refer to here as the Durbin term.

By accounting for this Durbin term, a SDPM can provide a more nuanced understanding of the spatial dynamics underlying the social or economic phenomenon being studied. This makes it a powerful tool for social science and regional economic research, as it can capture the intricacies of spatial dependence and interaction effects more effectively.

The following scripts demonstrate how to perform spatial Durbin panel model (non-spatial model), and the spatial lag Durbin and spatial error Durbin panel models, using the Greater Beijing Area data. As with before, you can alter the model , effect parameters to change between fixed, random, individual, time or two-ways effects.

# Spatial panel Durbin model:

# Create the spatial lag terms for the independent variables:

FININCPC.lag <- slag(spatial\_data\_p$FININCPC, gb.listw)

FDIPC.lag <- slag(spatial\_data\_p$FDIPC, gb.listw)

FIXINVPC.lag <- slag(spatial\_data\_p$FIXINVPC, gb.listw)

URB.lag <- slag(spatial\_data\_p$URB, gb.listw)

spatial\_data\_p\_dm <- data.frame (spatial\_data\_p, FININCPC.lag, FDIPC.lag, FIXINVPC.lag, URB.lag)

# Creating a pdata.frame

spatial\_data\_p\_dm <- pdata.frame(spatial\_data\_p\_dm, index = c("COUNTYNAME", "Year"))

gb.fm.spdm <- GDPPC ~ FININCPC + FDIPC + FIXINVPC + URB + FININCPC.lag + FDIPC.lag + FIXINVPC.lag + URB.lag

# Spatial Durbin model (non-spatial) fixed/random:

gb.spdm.fixed <- plm (gb.fm.spdm, data = spatial\_data\_p\_dm, model = "within", effect = "individual")

gb.spdm.random <- plm (gb.fm.spdm, data = spatial\_data\_p\_dm, model = "random", effect = "individual")

summary(gb.spdm.fixed)

summary(gb.spdm.random)

# Spatial lag/error Durbin model fixed/random:

gb.spdm.lag.fixed <- spml (gb.fm.spdm, data = spatial\_data\_p\_dm, listw = gb.listw, model = "within", effect = "individual", lag = TRUE, spatial.error = "none")

gb.spdm.error.fixed <- update(gb.spdm.lag.fixed, lag = FALSE, spatial.error = "kkp")

gb.spdm.lag.random <- update (gb.spdm.lag.fixed, model = "random")

gb.spdm.error.random <- update (gb.spdm.error.fixed, model = "random")

# Summarize the results:

summary(gb.spdm.lag.fixed)

summary(gb.spdm.error.fixed)

summary(gb.spdm.lag.random)

summary(gb.spdm.error.random)

Please check and compare the results very carefully. Pay attention to how the estimated coefficients change, how the rho and lambda values change in different model specifications, and how the significant levels of the estimated coefficients change as well. You are strongly encouraged to use your own data to repeat the same procedures and engage in the practices of analyzing the data and interpreting the data. If you want to further explore the Greater Beijing Area data and interpret the results from a regional studies perspective, please consult Yu and Wei (2008) for detailed background information and potential interpretation of the results.

10.3 Spatially varying coefficient process with panel model

As we have seen in Chapter 9, Spatially Varying Coefficient (SVC) models (represented by GWR and ESF SVC models) provide an effective means to address spatial heterogeneity, a characteristic of spatial data where the relationship between the dependent variable and independent variables can change across different spatial locations. SVC models are a flexible class of models that allow the coefficients of predictor variables to vary across space, enabling a very detailed understanding of spatial processes. In contrast to traditional fixed-coefficient models, SVC models do not assume that the influences of independent variables are constant over space, offering a more refined and context-specific understanding of the modeled relationships.

Implementing SVC models with panel data involves certain considerations. The key lies in recognizing that the relationships we are studying may not be homogenous across space, and instead may display region-specific behaviors. Given the time dimension in panel data, this means the relationships might not only be different from one location to another but can also evolve over time at each specific location.

There are a number of methods available to implement SVC models in a panel data context. I have developed in the early times a so-called homogenized geographically and temporally weighted regression (Yu, 2014). In which I attempted to treat temporal “space” as a similar to physical space, calling the action “homogenization.” However, while in practice homogenization is convenient and requires minimal changes to use with the existing spgwr package, the speculative nature of homogenization casts great doubt in broad application of this approach. Instead of further experimenting with homogenization of space and time, I have worked exclusively trying to integrate geographically weighted regression analysis with panel data analysis since 2009. I have later developed the geographically weighted panel regression approach (Yu, 2010) and applied it to study high-speed railway’s influence on China’s county development (Yu et al., 2021), and nonpharmaceutical interventions’ effect on the spread of COVID-19 in China (Yu et al., 2023). In an attempt to expand the ESF SVC model to the panel model, I have also attempted the approach in collaboration with the spmoran’s author, Dr. Daisuke Murakami, to investigate again high-speed rail’s influence on China’s county development (Yu et al., 2020). In this section, I will introduce these three relatively new approaches, but focus on ESF SVC, and geographically weighted panel regression, especially its implementation in R as an experimental practice.

Remember, the best approach to implement SVCs with panel data depends on the research question at hand and the characteristics of the data (this is especially true if one attempts the homogenized approach). It is important to consider the assumptions and requirements of each method, as well as their implications for the interpretation of the results. One must carefully check the model diagnostics and ensure the robustness of the findings. Ultimately, the SVC model offers a powerful tool to unveil and understand the complex spatial dynamics underlying the data.

10.3.1 Geographically and temporally weighted regression

Geographically and Temporally Weighted Regression (GTWR) is an extension of the Geographically Weighted Regression (GWR) model to panel data. It allows for both spatial and temporal non-stationarity in the relationships between independent and dependent variables to be modeled within the modeling framework.

The idea started with a very simple observation in that the essence of GWR is a local sample that was created based on the kernel function as we have seen in Chapter 9. For any kernel function, the parameter is a Euclidean distance and the constant bandwidth (which is essentially a Euclidean distance itself). As for in what dimensional spaces the Euclidean distance and bandwidth are calculated, the kernel function will act without change, and weigh observations the same, be it two-dimensional space (as in the cross-sectional GWR) or three-dimensional space (when temporal dimension is added).

Realizing this essence of the kernel function, I started an arbitrary “homogenization” treatment. In that treatment, I attempted to include the time axis as an additional dimension in generating a three-dimensional space and calculate Euclidean distance within it. Analogous to stacking the equations over time, this treatment essentially created a three-dimensional entity. The entity includes the usual planar coordinates and the time dimension. To avoid calculation traps, I arbitrarily make the time axis at the same measurement level as the planar coordinates by inflating the time coordinates by a factor of 10,000 (Yu, 2014). That is to say, the temporal “distance” from one year to another is 10,000 measuring unit as the planar coordinates. Or this is equivalent to say that the temporal association of one spatial unit over a year’s time has similar strength as 10,000 meters of spatial association. Apparently, this treatment is purely for convenient purposes.

This factor can be empirically derived, but in my experiments, I found the 10,000-inflation factor actually works pretty well, though it will certainly not be generalizable. I called this treatment “homogenization,” meaning homogenizing the space and time to consolidate a three-dimensional space for Euclidean distance calculation. After this treatment, all the data points in a panel data set hence have a distinct three-dimensional coordinate: the planar s and s, and the s for the time. In generating the kernel function and using it for creating local samples, it is this three-dimensional coordinate system that is used to calculate the distance, select a bandwidth within this three-dimensional space, and conduct GWR (basically still a cross-section GWR), hence the name homogenized geographically and temporally weighted regression.

This particular treatment does not require extra steps, but only changes the kernel weighting mechanism from two-dimensional space to three-dimensional space. The calibration procedure is identical to the cross-sectional GWR calibration and will not be repeated here. Interested readers can find detailed information in Yu (2014).

10.3.2 Geographically weighted panel regression

The GWPR analytical procedure (Yu, 2010) is an attempt to develop an exploratory spatial data analysis approach extended from cross-sectional geographically weighted regression (GWR) analysis (Fotheringham et al., 2002). The fundamental premise for GWR is that regressed relationships are not likely the same from place to place as suggested by conventional regression analysis because of different geographic backgrounds (including socioeconomic, cultural, demographic, and natural conditions). Through introducing a small bias, cross-sectional GWR analysis often reduces the variance of estimated coefficient quite significantly hence the analysis provides better confidence of the estimation (Fotheringham et al., 2002). The approach has seen wide application in many disciplines. Situating GW approach with the panel setting, however, is only explored in Yu (2010), Cai et al. (2014), and Yu et al. (2023).

To illustrate how geographically weighted panel regression can be estimated conceptually, let’s start with the most fundamental model specifications of the panel regression, extending from the general panel regression model as we presented in section 10.1.

Noted in that formulation, the coefficient vector **β** in equation does not change from location to location, hence the static model. The geographically weighted extension of the static panel model allows the coefficient vector **β** to change over locations (but remain constant over temporal periods for panel analysis). The geographically weighted panel regression model can then be written as:

() is the coordinate pairs of location . Everything else remains the same as in the general static panel regression model.

The introduction of the spatially varying coefficient immediately introduces either a **parsimonious** problem that we have more unknowns than data if the number of explanatory variables is more than the number of temporal periods, or the **collapse** of the panel data to a collection of individual time series estimation if we have longer time periods as in many social science studies with long panels.

In the parsimonious scenario, restrictions of the spatially varying mechanisms must be introduced for the coefficients to be estimable. This is what we have done in the cross-sectional GWR analysis by introducing the kernel function to mimic individual SDGP and weigh neighboring observations to create location specific local samples. I will not repeat the procedure here.

My experience with GWPR model development over the past decade points out that the real challenge is the collapse scenario. When I was working with short panels, as in Yu et al. (2021), I found the large number of spatial units and comparably small number of temporal periods (I have 2285 county units but only 9 time periods) allows a passable local panel sample to be created without trigger the temporal collapse. However, in Yu et al. (2023), the collapse problem prevents a meaningful GW approach to be applied because now we have a large number of temporal periods. In the collapse scenario, though the coefficients are now varying from place to place, the variation is not really “spatial” in the way a geographically weighted approach intends. After many experiments and testing, I found that the collapse problem only arises if we assume the geographical weighting schemes – the SDGPs that are represented by the kernel functions – remain temporally invariant. If, however, the geographical weighting schemes (SDGPs) are not assumed to be temporally invariant, we can then avoid the collapse scenario with geographically weighted panel regression. Because we are able to create local panel data instead of collapsed local temporal data sets. Details follow.

*Algorithm for* implementing *geographically weighted panel regression*

In a nutshell, geographically weighted approaches assume the observed data is generated by many overlapping and smooth spatial data generating processes that follow a distance-decaying mechanism (Fotheringham et al., 2002). This mechanism is a reflection of the First Law of Geography and is often mathematically represented by a kernel function and graphically a symmetric bell-shaped curve (Fotheringham et al., 2002). The observed data on location is hence the result of overlapping a smooth distance-decaying process that centered on location , and many other smooth distance-decaying processes that centered on other locations but are treated as neighbors of location in various degrees depending on the distance between those locations and location . We have discussed the geographical weighting in Chapter 9 in great detail and will not repeat it here.

The geographical weighting can solve the problem of the parsimonious scenario well because now for each location, there will be a weighted subsample that will have enough data to estimate the coefficients. “Geographical weighting” might also provide a more tenable way for estimation than simply relying on estimating non-related individual time series data on each location in the collapse scenario.

To effectively avoid the collapse scenario, however, we need to examine the geographical weighting in a more nuanced manner. In cross-sectional analysis, there is usually only one geographical weighting scheme representing the SDGP (one kernel function), though in the adaptive strategy, the bandwidth of the kernel function can change. In geographically weighted panel regression, however, we argue that the geographical weighting should be time variant and should be based on each cross-sectional dataset. This is to say that although the kernel function might still remain unchanged, the bandwidths (in the fixed strategy) or the numbers of nearest neighbors (in the adaptive strategy) do not have to stay constant over different temporal periods.

This is because although the geographical arrangement of observations does not change abruptly over the temporal period, assuming the “optimal” SDGPs remain the same for different temporal periods sounds less tenable. As an exploratory approach, a time varying spatial process might be closer to the true data generating process (DGP). It is very likely, however, that the time variant geographical weighting will generate different sizes of samples for different temporal periods. This will then create a subsample for each location to be an unbalanced panel dataset. Still, such treatment effectively avoids the collapse scenario when estimating geographically weighted panel regression.

The entire estimation procedure for the proposed geographically weighted panel regression follows these steps:

1. For each temporal period, the cross-sectional data is extracted. Cross-sectional geographically weighted regression approach is applied to this cross-section dataset to determine the geographically weighted subsample for each location at this temporal period. Specifically:

1.1 A distance-decaying spatial kernel function (Gaussian-like, bi-square or tri-cube) is chosen to decide the local region around a location and weigh the observations that fall within this local region. The weights will go through double weighting to ensure that all the weights assigned to one location will add up to one, as we have detailed in Chapter 9.

1.2 With the chosen spatial kernel function, a starting bandwidth is arbitrarily selected to determine the local region for each location. After double weighting, the weights are assigned for all the observations that fall within the local region using the kernel function with the arbitrarily chosen bandwidth . A local subsample for each location is created.

1.3 When all the locations have their own subsamples, ordinary least squares regression analysis at each location is conducted to produce the spatially varying coefficients of each explanatory variable.

1.4 An “optimal” bandwidth will be determined through optimization strategies that either maximize the model fit (such as the leave-one-out cross-validation approach) or minimize the information loss (such as the Akaike Information Criterion approach), or through genetic algorith by repeating steps 1.2 and 1.3 with different s.

1.5 Once an “optimal” is determined, the local region and weights for observations fall within this local region for all locations can be determined (the geographically weighted subsamples) for the specific temporal period.

2. For each location, the geographically weighted subsamples from all temporal periods are combined to be a (likely unbalanced) panel dataset.

3. For each location, regular panel regression analysis will be applied to estimate the coefficients of the explanatory variables.

4. The estimation will be repeated for all locations and the coefficients estimated in such a way will be spatially varying.

5. For each local panel regression, the regular statistical tests for the significance of the coefficients will also be produced. The significance test results will be used for mapping purposes.

I have developed a tentative R script to calibrate the GWPR model. The script has not been submitted to CRAN for packagin as when this book is written. It illustrates the fundamental ideas of GWPR as presented in this chapter. Below I will use the Greater Beijing Area’s data and the script to provide a basic GWPR analysis:

setwd("/yourworkspace")

# set the font for any output figures:

library(extrafont)

loadfonts(device = "win")

# set the font for any output figures:

windowsFonts(FontStyle=windowsFont("Times New Roman"))

# load necessary libraries:

library(spdep)

library(sp)

library(ggplot2)

library(spatstat)

library(classInt)

library(dplyr)

library(plm)

library(splm)

# Read the data:

spatial\_data <- st\_read(".", "gblntrans")

# Arrange spatial\_data by 'COUNTYNAME' alphabetically, this is critical for panel data analysis because the panel data transformation will re-arrange the spatial units alphabetically:

spatial\_data <- spatial\_data %>% arrange(COUNTYNAME)

spatial\_coord <- st\_coordinates(st\_centroid(spatial\_data))

# Import necessary packages

library(reshape2)

library(sf)

library(tidyverse)

# Convert sf object to data.frame

spatial\_data\_df <- as.data.frame(spatial\_data)

# Extract only the needed data for panel regression:

spatial\_data\_df <- spatial\_data\_df[, c("COUNTYNAME","GDPPC95","GDPPC96","GDPPC97","GDPPC98","GDPPC99","GDPPC00","GDPPC01",

"FININCPC95","FININCPC96","FININCPC97","FININCPC98","FININCPC99","FININCPC00","FININCPC01",

"FDIPC95","FDIPC96","FDIPC97","FDIPC98","FDIPC99","FDIPC00","FDIPC01",

"FIXINVPC95","FIXINVPC96","FIXINVPC97","FIXINVPC98","FIXINVPC99","FIXINVPC00","FIXINVPC01",

"URB95","URB96","URB97","URB98","URB99","URB00","URB01")]

# Reshape the data to long format

spatial\_data\_long <- spatial\_data\_df %>%

pivot\_longer(-COUNTYNAME,

names\_to = c(".value", "Year"),

names\_pattern = "([A-Za-z]+)([0-9]+)",

values\_drop\_na = TRUE)

# Correct the year values

spatial\_data\_long$Year <- ifelse(as.numeric(spatial\_data\_long$Year) < 50,

paste0("20", spatial\_data\_long$Year),

paste0("19", spatial\_data\_long$Year))

# Creating a pdata.frame

spatial\_data\_p <- pdata.frame(spatial\_data\_long, index = c("COUNTYNAME", "Year"))

# Set the formula

gb.fm.p <- GDPPC ~ FININCPC + FDIPC + FIXINVPC + URB

# Load the GWPR scripts:

source("GWPR\_unbalanced.r")

# Obtain the annually changing nearest neighbors using the adaptive strategy for each year, the search criterion is AIC:

gb.bd.p<-gwpr.bdwt(gb.fm.p, data = spatial\_data\_p, coords = spatial\_coord, method = "aic", gweight = gwr.adaptive)

# You can check the varying nearest neighbors by:

gb.bd.p

# Now calibrate the GWPR model, with individual fixed effects:

gb.gwpr<- gwpr.unbalanced(gb.fm.p, data = spatial\_data\_p, coords = spatial\_coord, nearestnb = gb.bd.p, effect = "individual", model = "within", inst.method = "baltagi")

# Map one of the spatially varying coefficients - FININCPC

# Convert row names to a column

gb.gwpr$gwpr.b$COUNTYNAME <- rownames(gb.gwpr$gwpr.b)

gb.gwpr$localpv$COUNTYNAME <- rownames(gb.gwpr$localpv)

# Join the dataframes

gb.df <- spatial\_data %>%

dplyr::left\_join(gb.gwpr$gwpr.b, by = "COUNTYNAME") %>%

dplyr::left\_join(gb.gwpr$localpv, by = "COUNTYNAME")

# Filter based on the p-value

gb.df$FININCPC.x[gb.df$FININCPC.y >=0.05] <-NA

# Now, make the map

p10.1<-ggplot() +

geom\_sf(data = gb.df, aes(fill = FININCPC.x)) +

theme\_minimal() +

scale\_fill\_gradient(low = "gray90", high = "black", name = "Financial Income\nPer capita", na.value = "white") +

labs(title = "Varying coefficients of Financial income per capita on GDP per capita") +

theme(text = element\_text(family = "Times New Roman"))

p10.1

ggsave(file ="figure10.1.jpg", p10.1, width = 7.25, height = 7, dpi = 600)

[Figure 10.1 is about here]

Just for comparison purposes, please start a new R session (do not work within the same R session as the GWPR codes) and use the following script to run a cross-sectional GWR analysis for year 2001, and produce the map for Financial Income per capita’s coefficients in 2001 (Figure 10.2). Compare the two maps to see the differences.

# Starting with a New R session:

# Do a cross sectional GWR analysis and compare:

setwd("/yourworkspace/")

# set the font for any output figures:

library(extrafont)

loadfonts(device = "win")

# set the font for any output figures:

windowsFonts(FontStyle=windowsFont("Times New Roman"))

# load necessary libraries:

library(spdep)

library(sp)

library(ggplot2)

library(spatstat)

library(spgwr)

spatial\_data <- st\_read(".", "gblntrans")

# construct the neighbors list

# Arrange spatial\_data by 'COUNTYNAME' alphabetically, this is critical for panel data analysis because the panel data transformation

# Will re-arrange the spatial units alphabetically:

spatial\_data <- spatial\_data %>% arrange(COUNTYNAME)

spatial\_coord <- st\_coordinates(st\_centroid(spatial\_data))

gb.fm <- GDPPC01 ~ FININCPC01 + FDIPC01 + FIXINVPC01 + URB01

gb.bd <- gwr.sel(gb.fm, data = spatial\_data, coords = spatial\_coord, adapt = T, gweight = gwr.bisquare, method = "aic")

gb.gwr.crs <- gwr(gb.fm, data = spatial\_data, coords = spatial\_coord, gweight = gwr.bisquare, adapt = gb.bd)

gb.out <- st\_as\_sf(gb.gwr.crs$SDF)

gb.out <- st\_set\_crs(gb.out, st\_crs(spatial\_data)) # Make sure the coordinate systems are the same

gb.out.poly <- st\_join(spatial\_data, gb.out, join = st\_contains) # Spatial Join

# Map the Financial Income per capita:

p10.2<-ggplot() +

geom\_sf(data = gb.out.poly, aes(fill = FININCPC01.y)) +

scale\_fill\_gradient(low = "gray90", high = "black") +

theme\_minimal() +

labs(title = "Varying coefficients of Financial Income per capita on \nGDP per capita 2001", fill = "Financial Income\nPer capita") +

theme(text = element\_text(family = "Times New Roman"))

p10.2

ggsave (file ="figure10.2.jpg", p10.2, width = 7.25, height = 7, dpi = 600)

The GWPR map (Figure 10.1) shows a stronger variation of the coefficients than the cross-sectional GWR map (Figure 10.2). This is to be expected since the panel dataset has more degrees of freedom than the cross-sectional dataset. The local nuances are more readily revealed. Still, when interpreting GW results, we shall pay more attention to the spatial patterns of the varying coefficients than the actual values to make the most of the exploratory nature of GW family of analysis.

10.3.3 Random effect eigenfunction based spatial filtering SVC panel regression

Random Effect Eigenfunction Based Spatial Filtering SVC Panel Regression (REF-SF-SVC-PR) uses random effect models with spatial filters to capture spatially varying relationships. This is an alternative to the GW family of models. The SVC implementation for cross-sectional data has been implemented in the spmoran package and demonstrated in Chapter 9. Implementation of the ESF SVC model to panel dataset, however, has only been discussed in Yu et al. (2020). The following discussion is a replication of what appears in Yu et al. (2020). The technical details, however, are provided here only for reference purposes. Technical savvy readers can read through to have a thorough understanding, but the application of the ESF SVC panel regression does not require full comprehension of these details. The R scripts are implemented and will be provided as the companion website just as the GWPR scripts.

A panel model considering residual spatial dependence is readily constructed by combining the ESF model and the panel model as follows (we are using the full temporal span to refer the variable vectors and matrices):

|  |  |
| --- | --- |
|  |  |

where is an matrix that stacks the matrix (the selected eigenvectors formed matrix) times. This equation assumes the same spatial process , over time with random effects assumed on the coefficients. In short, this model describes spatial autocorrelation effects, , as well as the individual/group-specific effects, , and the temporal effects, , respectively**.**

Following Murakami et al. (2017), SVCs are readily introduced as follows:

|  |  |
| --- | --- |
|  | (8) |

where is theelement-wise product operator, is the vector of th explanatory variable, and is the corresponding SVC specified as:

|  |  |
| --- | --- |
|  | (9) |

is the mean of the th SVC. The spatial variation around the mean is modeled using the spatial process depending on the variance and scale parameters (, ). Unlike the basic geographically weighted regression as we discussed in Chapter 9, the spatial scale is estimated for individual SVCs here by estimating , which determines , just like the multiscale GWR (Fotheringham et al., 2017). Unfortunately, the above two equations include variance parameters that must be numerically estimated if and are fixed coefficients while if they are random. This could be rather computationally intensive. A fast computation strategy is developed in Murakami and Griffith (2019a)

Note that the ESF panel model with SVC has the following expression:

|  |  |
| --- | --- |
|  |  |

where , , is a block diagonal matrix with the first block being and the *k*-th block being , and (*τ*2, *τ*12, …, *τK2*, *α*, *α1*, …, *αK*). This expression is analytically identical to the ESF panel regression; they can be estimated in similar ways that we will detail below.

*Estimation under fixed and*

It is very important to note that the ESF panel model with SVCs is not available if is fixed and dummies are given for individual geographical units as usual. This is because if each eigenvector in isregressed on , the corresponding residuals that are used in the least squares dummy variables (LSDV) estimation that are the standard fixed effects panel estimation approach are uniformly zero. It occurs because these regressions attempt to model each eigenvector, which has unique values at the maximum, using regression coefficients and an intercept. It means that all the residual spatial variations are absorbed by the fixed individual effects and the spatial filtering term is no longer needed in this specification.

On the other hand, the model with SVCs is available as long as the value of explanatory variables changes depending on time. Estimation procedures follow:

1. Regress and each of the variables in on [, ] to obtain residuals and , where .
2. Regress on . This step eliminates the and terms,
3. The resulting model here:

|  |
| --- |
|  |

is identical to the Moran eigenvector-based SVC model of Murakami et al. (2017) without spatially varying intercept filtering residual spatial dependence. Thus, all the parameters are estimated by applying their Type II restricted likelihood estimation method, which is fast and applicable even for millions of samples (Murakami and Griffith, 2019b).

*Estimation under random and*

In this case, our model is identical to the SVC model with group effects whose fast Type II restricted likelihood estimation is explained in Murakami and Griffith (2019a). Their Type II maximization can be viewed as an empirical Bayes approach estimating the random effects model, including the fixed effects model as a special case, by balancing the trade-off between bias and variance (similar to GWR approach). In addition, the random specification enables us to identify the autocorrelated variations and independent/group specific variations behind residuals. The random effects specification is more attractive than the fixed effects one to identify spatial and non-spatial effects while the fixed effects specification is a faster and practical alternative.

The following R scripts demonstrate in a very detailed way how the random eigenfunction spatial filtering spatially varying coefficient models (RESF SVC) are estimated in R. The scripts for estimation are available from the book’s companion website:

# The next block of codes attempts to evaluate the varying coefficients through RESF\_VC routine:

library(vegan)

library(fields)

# These source codes are available from the book’s companion website.

source("./resf/resf\_panel.R")

source("./resf/resf\_vc\_panel.R")

source("./resf/resf\_p.R")

source("./resf/resf\_vc\_p.R")

source("./resf/meigen2.R")

source("./resf/meigen\_f2.R")

# Extract the spatial unit ID and time ID, this is required for resf panel vc model.

s\_id <- spatial\_data\_p$COUNTYNAME

t\_id <- spatial\_data\_p$Year

# Extract the dependent and independent variables:

lmod <- lm(gb.fm.p, data = spatial\_data\_p)

dep <- lmod$model[, 1]

indp <- lmod$model[, -1]

# The ids for each spatial units used to construct the Eigenvector:

cid<-seq(1,dim(spatial\_coord)[1])

# All coefficients are assumed to be spatially varying. If there is strong theoretical guidance that some of the independent variables are not spatially varying, then they need to be specified using "xconst =" argument

gb.meig <-meigen2(coords=spatial\_coord, s\_id=cid) ## Moran's eigenvectors

################ fixed effect spatially varying coefficient model (the result could be unstable because the individual effects might very well absorb the spatially varying effect.

gb.resf.svc.fe<- resf\_vc\_panel(y=dep, x=indp, s\_id=s\_id, t\_id=t\_id, meig=gb.meig , pmodel="within", effect="individual")

################ random effects spatially varying model (I personally recommend these models, which seem stable)

gb.resf.svc.rd<- resf\_vc\_panel(y=dep, x=indp,s\_id=s\_id, t\_id=t\_id, meig=gb.meig, pmodel="random",effect="individual")

# First check to see which variables are actually varying across space, and which are not:

gb.resf.svc.fe$vc

gb.resf.svc.rd$vc

# 1 represents the coefficients of the variable is spatially varying, and 0 is not.

# Now extract the spatially varying coefficients and corresponding p-values:

# Because of how the RESF SVC model's output structure, the output coefficients and p-values are identical set stacked over the temporal periods.

# That means only the first time periods's values need to be extracted:

gb.out.fe.b <- gb.resf.svc.fe$b\_vc [1: length(cid),]

gb.out.fe.p <- gb.resf.svc.fe$p\_vc [1: length(cid),]

gb.out.rd.b <- gb.resf.svc.rd$b\_vc [1: length(cid),]

gb.out.rd.p <- gb.resf.svc.rd$p\_vc [1: length(cid),]

# Map the varying coefficients of FININCPC:

# Rename the columns

names(gb.out.fe.p)[names(gb.out.fe.p) == "FININCPC"] <- "FININCPC.p"

names(gb.out.fe.b)[names(gb.out.fe.b) == "FININCPC"] <- "FININCPC.b"

names(gb.out.rd.p)[names(gb.out.rd.p) == "FININCPC"] <- "FININCPC.p"

names(gb.out.rd.b)[names(gb.out.rd.b) == "FININCPC"] <- "FININCPC.b"

# Combine data by columns

spatial\_data.fe <- st\_bind\_cols(spatial\_data, gb.out.fe.p["FININCPC.p"], gb.out.fe.b["FININCPC.b"])

spatial\_data.rd <- st\_bind\_cols(spatial\_data, gb.out.rd.p["FININCPC.p"], gb.out.rd.b["FININCPC.b"])

spatial\_data.fe$FININCPC.b[spatial\_data.fe$FININCPC.p >=0.05] <-NA

spatial\_data.rd$FININCPC.b[spatial\_data.rd$FININCPC.p >=0.05] <-NA

# Now, make the maps

p10.3<-ggplot() +

geom\_sf(data = spatial\_data.fe, aes(fill = FININCPC.b)) +

theme\_minimal() +

scale\_fill\_gradient(low = "gray90", high = "black", name = "Financial Income\nPer capita", na.value = "white") +

labs(title = "Varying coefficients of Financial income per capita \non GDP per capita based on RESF fixed effect model") +

theme(text = element\_text(family = "Times New Roman"))

p10.3

ggsave(file ="figure10.3.jpg", p10.3, width = 7.25, height = 7, dpi = 600)

p10.4<-ggplot() +

geom\_sf(data = spatial\_data.rd, aes(fill = FININCPC.b)) +

theme\_minimal() +

scale\_fill\_gradient(low = "gray90", high = "black", name = "Financial Income\nPer capita", na.value = "white") +

labs(title = "Varying coefficients of Financial income per capita \non GDP per capita based on RESF random effect model") +

theme(text = element\_text(family = "Times New Roman"))

p10.4

ggsave(file ="figure10.4.jpg", p10.4, width = 7.25, height = 7, dpi = 600)

[Figures 10.3 and 10.4 are about here]

You are strongly encouraged to compare the four figures produced in this chapter to have a sense of how these models can facilitate understanding your data and build an appropriate model to tell the story. For background information about the Greater Beijing Area and its regional economic development dynamics, please consult Yu and Wei (2008) for a better interpretation of these four figures. Still, you are strongly encouraged to use your own data and replicate what I have presented in this chapter and attempt to tell your own story.

10.4 Practice in R

In this section, I want you to collect your own data (starting from many shapefiles with multiple time periods’ information, or one shapefile with multiple time periods’ information as separate attributes), then follow step by step the examples I presented in this chapter using the R-scripts and the source codes. These hands-on experiences will deepen your understanding of spatial panel regression and SVC panel regression (including GWPR and RESF SVC models). This practical understanding will, in turn, significantly amplify your capability to employ these methods in your academic inquiries or professional assignments.

Practicing with your own data will be critical for you to have a practical understanding of the concepts and methods introduced in this chapter. Some of the narratives in this chapter might be too technical and involve too much linear algebraic manipulation (especially the RESF SVC section). As a social scientist practitioner, however, while understanding the fundamental mechanisms of the models is important, it is equally, if not more, important to focus on the applicability of these models to your specific context.

Consequently, replicating the scripts outlined in this chapter with your own data is an indispensable step. This experience will not only enable you to master spatial data analysis, but also help you utilize these methods as powerful tools for storytelling. By doing so, you will be paving the way for breakthroughs in your domain knowledge.

So, set out on this enriching journey. Embrace the complexities, the learning, and the opportunities for growth. Your mastery over these analytical tools is not just about enhancing your expertise, it is also about amplifying the impact of your research or work, thereby contributing meaningfully to the world of knowledge. This journey, filled with practical experiences and applied learning, is certain to prove invaluable in your progression as a researcher and practitioner.

*Review Questions*

1. **Question:** Briefly explain the importance of spatial panel data in social science research.

**Answer:** Spatial panel data allows for a deeper understanding of social phenomena by accounting for both spatial relationships and temporal dynamics. It allows researchers to investigate spatio-temporal patterns and trends, quantify spatial spillover effects, and understand spatial heterogeneity.

1. **Question:** What is a spatial panel model and how does it differ from traditional panel data models?

**Answer:** A spatial panel model is a type of statistical model used when dealing with spatial and temporal data. Unlike traditional panel models, spatial panel models incorporate spatial lags of the dependent variable, the independent variables, or the error term to account for spatial correlation and spillover effects.

1. **Question:** What does the term "spillover effects" mean in the context of spatial data analysis?

**Answer:** In spatial data analysis, "spillover effects" refer to the influence or impact of an event or policy in one location on neighboring locations. These effects represent the spatial interdependencies often seen in geographic data.

1. **Question:** What is the purpose of using Space-Varying Coefficient (SVC) models in spatial panel data analysis?

**Answer:** SVC models are used to account for spatial heterogeneity in the relationship between the dependent and independent variables. They allow for the coefficients of the regression model to vary across space, offering a more nuanced understanding of the relationships in the data.

1. **Question:** What are some methods to visualize the results of SVC panel regressions? **Answer:** Results from SVC panel regressions can be visualized using maps to display how coefficients vary spatially. Other visualization techniques may include line graphs or scatter plots to represent temporal changes.
2. **Question**: What is the Geographically Weighted Panel Regression (GWPR) model and what are the two possible problems when estimating GWPR models and how are they resolved?

**Answer**: The GWPR model is an extension of the cross-sectional GWR model that is developed to apply specifically to panel data sets. There are usually two possible problems when estimating GWPR models. The first is the parsimonious problem, meaning there are more parameters to be estimated than there are data. This is solved by applying the geographical weighting. The second problem is the potential collapse problem, especially when there are many temporal periods. Collapse problem eventually reduces the panel regression to a series of unrelated time series analysis. It is solved by estimating the optimal bandwidths (or nearest neighbors) for different temporal periods differently.

1. **Question:** What is the use of LM test in spatial panel data analysis?

**Answer:** The LM test is used to test for possible sources of the spatial autocorrelation in the regression residuals. There are four types of LM tests, spatial lag, spatial error, locally robust spatial lag, and locally robust spatial error LM tests. Usually, the one test that suggests the least likelihood of making a Type I error against the Null Hypotheses (no spatial lag under the condition of no spatial error, no spatial error under the condition of no spatial lag, no spatial lag under the condition of spatial error, and no spatial error under the condition of spatial lag) will be selected as the most appropriate model.

1. **Question:** What is the difference between a fixed effects model and a random effects model in the context of spatial panel data?

**Answer:** A fixed effects model allows for individual-specific effects that are correlated with the independent variables, while a random effects model assumes these effects are uncorrelated. The choice between these models depends on the nature of the data and the research question.

1. **Question:** Why are spatial panel or SVC panel analyses more complex?

**Answer:** The complexity of spatial panel or SVC panel analyses results from the many different types of specifications that the panel data can be formulated to. The unobservable effects can be assumed to be fixed or random, and the effects can also be individual specific, temporal period specific or both, which essentially create not one, but a host of models that could potentially produce many different results. It is hence critical to have a thorough understanding of one’s data, theoretical narratives, and research design and purposes.

1. **Question:** How does time-varying spatial weight contribute to GWPR analysis?

**Answer:** Time-varying spatial weights can capture changing spatial relationships over time. This is very important to address the collapse problem in GWPR analysis. More importantly, it is also more realistic to assume the spatial data generating process (SDGP) will not stay constant over time, but evolves based on time-specific factors, such as political environment, investment fluctuations, technological advancements, etc.

1. **Question:** What is a spatial error model (SEM) and when might it be appropriate to use one?

**Answer:** A SEM includes a spatially lagged error term to account for spatial autocorrelation. This model is appropriate when spatial dependence is believed to arise from omitted variables that are spatially autocorrelated, or what we termed the correlated effects.

1. **Question:** In the context of spatial data, what does the term "spatial autocorrelation" refer to? **Answer:** Spatial autocorrelation refers to the degree to which a variable of interest is correlated with itself in space. In other words, it reflects the similarity of values between geographically close observations.
2. **Question:** What is the Random Effect Spatial Filtering Space-varying Coefficient (RESF SVC) model and how does it differ from a traditional random effects model?

**Answer:** The RESF SVC model is a variant of the traditional random effects model that allows for spatially varying coefficients. Unlike the traditional model, the RESF SVC can capture the spatial heterogeneity in the relationships between the dependent and independent variables.

1. **Question:** Why is understanding spatial heterogeneity crucial in spatial panel data analysis?

**Answer:** Spatial heterogeneity acknowledges that relationships between variables may not be constant across space. Understanding this can lead to more accurate models and prevent misleading inferences, especially when the analysis intends to provide more detailed understanding of the phenomena under study or intend to provide targeted policies for different regions in the research area.

1. **Question:** What is a pooled OLS model, and when is it appropriate to use in spatial panel data analysis?

**Answer:** A pooled OLS model combines cross-sectional and time series data but ignores the potential impacts of individual and temporal effects. It is often used as a starting point or baseline model in spatial panel data analysis. It can also be used to test the existence of either individual or time or both effects.

1. **Question:** What are some of the challenges in implementing GWPR and RESF SVC models?

**Answer:** Implementing GWPR and RESF SVC models can be computationally intensive, especially with large datasets. Interpreting the results can also be more complex given the local nature of the estimates and the complexity of not one model, but a cohort of models that we need to carefully sift through to hope for revealing what the data intends to tell us the best.

1. **Question:** What are some common issues that arise when working with spatial panel data and how can they be addressed?

**Answer:** Common issues can include non-stationarity, spatial autocorrelation, and spatial heterogeneity. These can be addressed using techniques such as differencing for stationarity, spatial regression models for autocorrelation, and SVC models for heterogeneity.

1. **Question:** How can the results of a GWPR or a RESF SVC model be visually represented?

**Answer:** The results of GWPR and RESF SVC models can be visually represented using maps to show how the coefficients vary across space. More specifically, the varying coefficients can be displayed in combination with their corresponding varying p-values. With only the significant ones displayed, the map provides a powerful visualization tool for scholars to interpret the local patterns more efficiently. This can provide a more intuitive understanding of the spatial patterns in the data.

1. **Question:** In the process of geographically weighted regression, we mention “kernel functions.” What role do they play in the GWPR model?

**Answer:** Kernel functions are used to assign weights to the observations based on their spatial proximity to the location where the model is being estimated. It is a quantification tool for the spatial data generating process (SDGP). These weights are used in the regression, with closer observations having more influence on the parameter estimation.

1. **Question:** What are the advantages of implementing Space-Varying Coefficient (SVC) models?

**Answer:** SVC models capture spatial heterogeneity in the relationships between variables, enhancing the understanding of the spatial processes. They also enable the investigation of how the effects of certain factors change across space, adding a more detailed layer of insight to spatial data analysis.

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